NOIP 专题训练：数据结构（二）

【注意事项】

- 不可以使用赛前已经有的代码。
- 不需要文件输入输出。
- 训练赛采用 IOI 赛制。每道题最多提交 32 次。
- 测试环境为 Linux，编译命令为 -std=c++14,-O2。
- 祝选手好运。
- 比赛由窗社（*mado-soft*）精神赞助！

【题目列表】

英文名称	时间限制	空间限制	码长限制	题目类型	提交连接
DFS	6s	500MB	20KB	传统	submit
Nastya and CBS	4s	250MB	20KB	传统	submit
Andryusha and Nervous Barriers	4s	250MB	20KB	传统	submit
Jumping Through the Array	8s	500MB	20KB	传统	submit
Iqea	3s	250MB	20KB	传统	submit

【赞助商广告】

和泉妃爱：哥哥把很棒的女性把到手了。恭喜，哥哥！

Problem A. DFS

【题目描述】

Let T be a tree on n vertices. Consider a graph G_0 , initially equal to T . You are given a sequence of q updates, where the i -th update is given as a pair of two distinct integers u_i and v_i .

For every i from 1 to q , we define the graph G_i as follows:

- If G_{i-1} contains an edge $\{u_i, v_i\}$, then remove this edge to form G_i .
- Otherwise, add this edge to G_{i-1} to form G_i .

Formally, $G_i := G_{i-1} \triangle \{\{u_i, v_i\}\}$ where $\triangle$ denotes the set *symmetric difference*.

Furthermore, it is guaranteed that T is always a subgraph of G_i . In other words, an update never removes an edge of T .

Consider a connected graph H and run a depth-first search on it. One can see that the tree edges (i.e. the edges leading to a not yet visited vertex at the time of traversal) form a spanning tree of the graph H . This spanning tree is not generally fixed for a particular graph — it depends on the starting vertex, and on the order in which the neighbors of each vertex are traversed.

We call vertex w good if one can order the neighbors of each vertex in such a way that the depth-first search started from w produces T as the spanning tree. For every i from 1 to q , find and report the number of good vertices.

【输入格式】

The first line contains two integers n and q ($3 \le n \le 2 \cdot 10^5$, $1 \le q \le 2 \cdot 10^5$) — the number of nodes and the number of updates, respectively.

Each of the next $n-1$ lines contains two integers u and v ($1 \le u, v \le n$, $u \ne v$) — vertices connected by an edge in T . It is guaranteed that this graph is a tree.

Each of the next q lines contains two integers u and v ($1 \le u, v \le n$, $u \ne v$) — the endpoints of the edge that is added or removed. It is guaranteed that this edge does not belong to T .

【输出格式】

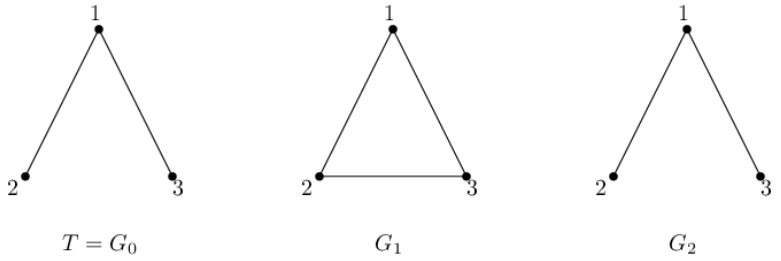
For each update, print one integer k — the number of good vertices w after the corresponding update.

【测试样例】

样例输入	样例输出
3 2 1 2 1 3 2 3 3 2	2 3
6 6 1 2 2 3 1 4 4 5 1 6 2 5 3 4 5 2 6 4 3 4 6 5	3 2 3 2 3 2

【样例解释】

The first sample is depicted in the following figure.



After the first update, G contains all three possible edges. The result of a DFS is as follows:

- Let the starting vertex be 1 . We have two choices of ordering the neighbors of 1 , either [2,3] or [3,2] .
  - If we choose the former, then we reach vertex 2 . Regardless of the ordering of its neighbors, the next visited vertex will be 3 . Thus, the spanning tree generated by this DFS will contain edges {1,2} and {2,3} , which does not equal to T .
  - If we choose the latter, we obtain a spanning tree with edges {1,3} and {2,3} .

  Hence, there is no way of ordering the neighbors of vertices such that the DFS produces T , and subsequently 1 is not a good vertex.
- Let the starting vertex be 2 . We have two choices of traversing its neighbors. If we visit 3 first, then the spanning tree will consist of edges {2,3} and {1,3} , which is not equal to T . If we, however, visit 1 first, then we can only continue to 3 from here, and the spanning tree will consist of edges {1,2} and {1,3} , which equals to T . Hence, 2 is a good vertex.
- The case when we start in the vertex 3 is symmetrical to starting in 2 , and hence 3 is a good vertex.

Therefore, the answer is 2 .After the second update, the edge between 2 and 3 is removed, and $G = T$. It follows that the spanning tree generated by DFS will be always equal to T independent of the choice of the starting vertex. Thus, the answer is 3 .

In the second sample, the set of good vertices after the corresponding query is:

- {2,3,5}
- {3,5}
- {3,4,5}
- {4,5}
- {4,5,6}
- {5,6}

Problem B. Nastya and CBS

【题目描述】

Nastya is a competitive programmer, but she is only studying now. Recently, Denis told her about the way to check if the string is correct bracket sequence. After that, unexpectedly, Nastya came up with a much more complex problem that Denis couldn't solve. Can you solve it?
A string s is given. It consists of k kinds of pairs of brackets. Each bracket has the form t — it is an integer, such that $1 \leq |t| \leq k$. If the bracket has a form t , then:

- If $t > 0$, then it's an opening bracket of the type t .
- If $t < 0$, then it's a closing bracket of the type $-t$.

Thus, there are k types of pairs of brackets in total.
The queries need to be answered:

1. Replace the bracket at position i with the bracket of the form t .
2. Check if the substring from the l -th to r -th position (including) is the correct bracket sequence.

Recall the definition of the correct bracket sequence: - An empty sequence is correct.

- If A and B are two correct bracket sequences, then their concatenation " A B " is also correct bracket sequence.
- If A is the correct bracket sequence, c $(1 \leq c \leq k)$ is a type of brackets, then the sequence "c A $-c$" is also correct bracket sequence.

【输入格式】

The first line contains an integer n $(1 \leq n \leq 10^5)$ — length of string and k $(1 \leq k \leq n)$ — the number of kinds of pairs of brackets.
The second line contains string s of length n — n integers $s_1, s_2, ..., s_n$ $(1 \leq |s_i| \leq k)$
The third line contains single integer q $(1 \leq q \leq 10^5)$ — the number of queries.
Each of the following q lines describes the queries:

- 1 i t - query of the 1 type $(1 \leq i \leq n, 1 \leq |t| \leq k)$.
- 2 l r - query of the 2 type $(1 \leq l \leq r \leq n)$.

【输出格式】

For each query of 2 type, output "Yes" if the substring from the query is the correct bracket sequence and "No", otherwise.
All letters can be displayed in any case.

【测试样例】

样例输入	样例输出
2 1 1 -1 1 2 1 2	Yes
2 2 1 -2 1 2 1 2	No
6 2 1 2 -2 -1 1 -1 3 2 1 6 2 1 4 2 2 5	Yes Yes No
2 2 -1 1	No Yes

样例输入	样例输出
(continue) 4 2 1 2 1 1 1 1 2 -1 2 1 2	

【样例 4 解释】

In the fourth test, initially, the string is not a correct bracket sequence, so the answer to the first query is “No“. After two changes it will be equal to “ 1 −1 “, so it is a correct bracket sequence and the answer to the fourth query is “Yes“.

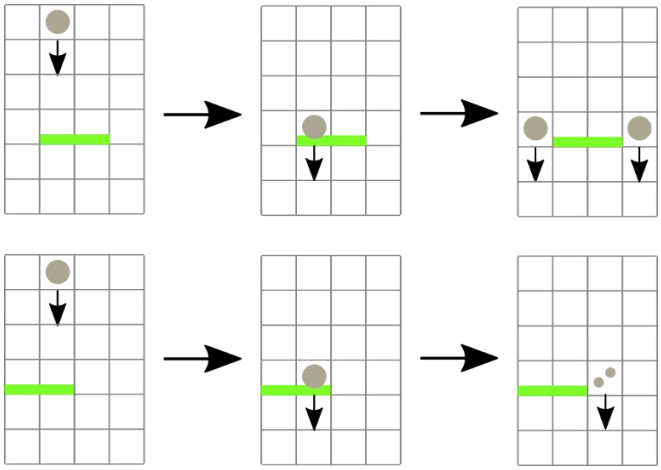
Problem C. Andryusha and Nervous Barriers

【题目描述】

Andryusha has found a perplexing arcade machine. The machine is a vertically adjusted board divided into square cells. The board has w columns numbered from 1 to w from left to right, and h rows numbered from 1 to h from the bottom to the top.

Further, there are barriers in some of board rows. There are n barriers in total, and i -th of them occupied the cells l_i through r_i of the row u_i . Andryusha recollects well that no two barriers share the same row. Furthermore, no row is completely occupied with a barrier, that is, at least one cell in each row is free.

The player can throw a marble to any column of the machine from above. A marble falls downwards until it encounters a barrier, or falls through the bottom of the board. A marble disappears once it encounters a barrier but is replaced by two more marbles immediately to the left and to the right of the same barrier. In a situation when the barrier is at an edge of the board, both marbles appear next to the barrier at the side opposite to the edge. More than one marble can occupy the same place of the board, without obstructing each other's movement. Ultimately, all marbles are bound to fall from the bottom of the machine.



Examples of marble-barrier interaction.

Peculiarly, sometimes marbles can go through barriers as if they were free cells. That is so because the barriers are in fact alive, and frightened when a marble was coming at them from a very high altitude. More specifically, if a marble falls towards the barrier i from relative height more than s_i (that is, it started its fall strictly higher than $u_i + s_i$), then the barrier evades the marble. If a marble is thrown from the top of the board, it is considered to appear at height $(h + 1)$.

Andryusha remembers to have thrown a marble once in each of the columns. Help him find the total number of marbles that came down at the bottom of the machine. Since the answer may be large, print it modulo $10^9 + 7$.

【输入格式】

The first line contains three integers h , w , and n ($1 \le h \le 10^9$, $2 \le w \le 10^5$, $0 \le n \le 10^5$) — the number of rows, columns, and barriers in the machine respectively.

Next n lines describe barriers. i -th of these lines containts four integers u_i , l_i , r_i , and s_i ($1 \le u_i \le h$, $1 \le l_i \le r_i \le w$, $1 \le s_i \le 10^9$) — row index, leftmost and rightmost column index of i -th barrier, and largest relative fall height such that the barrier does not evade a falling marble. It is guaranteed that each row has at least one free cell, and that all u_i are distinct.

【输出格式】

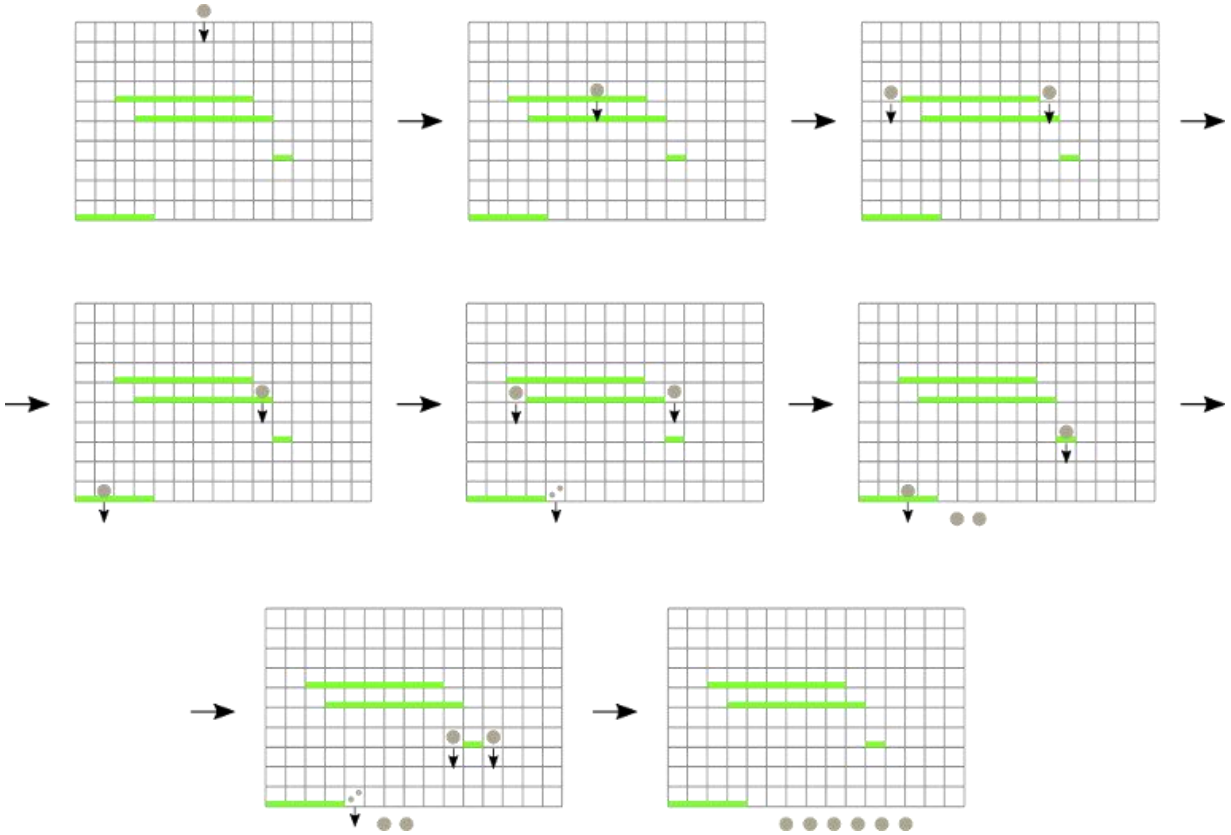
Print one integer — the answer to the problem modulo $10^9 + 7$.

【测试样例】

样例输入	样例输出
10 5 1 3 2 3 10	7
10 5 2 3 1 3 10 5 3 5 10	16
10 5 2 3 1 3 7 5 3 5 10	14
10 15 4 7 3 9 5 6 4 10 1 1 1 4 10 4 11 11 20	53

【样例解释】

In the first sample case, there is a single barrier: if one throws a marble in the second or the third column, two marbles come out, otherwise there is only one. The total answer is 7 .
In the second sample case, the numbers of resulting marbles are 2 , 2 , 4 , 4 , 4 in order of indexing columns with the initial marble.
In the third sample case, the numbers of resulting marbles are 1 , 1 , 4 , 4 , 4 . Note that the first barrier evades the marbles falling from the top of the board, but does not evade the marbles falling from the second barrier.
In the fourth sample case, the numbers of resulting marbles are 2 , 2 , 6 , 6 , 6 , 6 , 6 , 6 , 6 , 1 , 2 , 1 , 1 , 1 , 1 . The picture below shows the case when a marble is thrown into the seventh column.



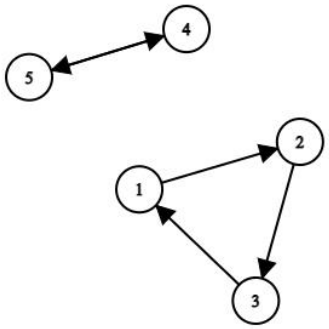
The result of throwing a marble into the seventh column.

Problem D. Jumping Through the Array

【题目描述】

You are given an array of integers a of size n and a permutation p of size n . There are q queries of three types coming to you:

1. For given numbers l and r , calculate the sum in array a on the segment from l to r.
2. You are given two numbers v and x . Let's build a directed graph from the permutation p : it has n vertices and n edges $i \to p_i$. Let C be the set of vertices that are reachable from v in this graph. You should add x to all a_u such that u is in C .
3. You are given indices i and j . You should swap p_i and p_j .



The graph corresponding to the permutation [2,3,1,5,4].

Please, process all queries and print answers to queries of type 1 .

【输入格式】

The first line contains a single integer n ($1 \le n \le 2 \cdot 10^5$) — the size of the array and permutation.

The second line contains n integers $a_1, a_2, ..., a_n$ ($-10^8 \le a_i \le 10^8$).

The third line contains n distinct integers $p_1, p_2, ..., p_n$ ($1 \le p_i \le n$).

The fourth line contains a single integer q — the number of queries ($1 \le q \le 2 \cdot 10^5$).

Next q lines contain description of queries. The i -th of them starts with an integer t_i ($1 \le t_i \le 3$) — the query type.

- If $t_i = 1$, then the i -th line also contains two integers l , r ($1 \le l \le r \le n$).
- If $t_i = 2$, then the i -th line also contains two integers v , x ($1 \le v \le n$, $-10^8 \le x \le 10^8$).
- If $t_i = 3$, then the i -th line also contains also two integers i , j ($1 \le i, j \le n$).

【输出格式】

For every first type query, print a single integer — the answer to this query.

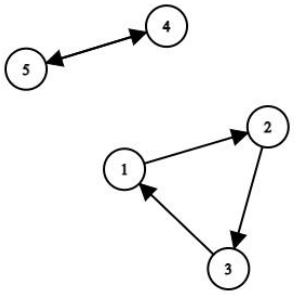
【测试样例】

样例输入	样例输出
5 6 9 -5 3 0 2 3 1 5 4 6 1 1 5 2 1 1 1 1 5 3 1 5 2 1 -1 1 1 5	13 16 11

样例输入	样例输出
8 -15 52 -4 3 5 9 0 5 2 4 6 8 1 3 5 7 10 2 2 2 2 5 -1 1 1 8 1 1 5 1 5 8 3 1 6 2 1 50 1 1 8 2 6 -20 1 1 8	61 45 22 461 301
1 1 1 1 1 1 1	1

【样例解释】

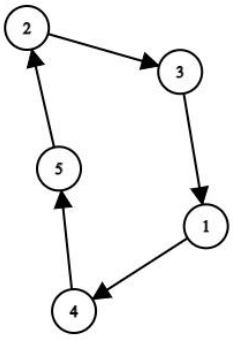
In the first example:



The graph corresponding to the initial permutation.

There are 6 queries.

1. The sum on the segment from 1 to 5 is $a_1 + a_2 + a_3 + a_4 + a_5 = 6 + 9 + (-5) + 3 + 0 = 13$.
2. If we start from 1 , we can reach $\{1,2,3\}$. After this query a is: $[7,10,-4,3,0]$.
3. The sum on the segment from 1 to 5 is $a_1 + a_2 + a_3 + a_4 + a_5 = 6 + 9 + (-5) + 3 + 0 = 16$.
4. After this query $p = [4,3,1,5,2]$.



The graph corresponding to the new permutation.

5. If we start from 2 , we can reach $\{1,2,3,4,5\}$. After this query a is: $[6,9,-5,2,-1]$.
6. The sum on the segment from 1 to 5 is $a_1 + a_2 + a_3 + a_4 + a_5 = 6 + 9 + (-5) + 2 + (-1) = 11$.

Problem E. Iqea

【题目描述】

Gridland is placed on infinite grid and has a shape of figure consisting of cells. Every cell of Gridland is a city. Two cities that are placed in adjacent cells are connected by the road of length 1 . It's possible to get from any city to any other city using roads. The distance between two cities is the minimum total road length on path from one city to another. It's possible to get from any cell that doesn't belong to Gridland to any other cell that doesn't belong to Gridland by using only cells which don't belong to Gridland. In other words, Gridland is connected and complement of Gridland is also connected.

At the moment no city in Gridland has Iqea famous shop. But Iqea has great plans for building shops in Gridland. For customers' convenience Iqea decided to develop an application. Using this application everyone can learn the distance to the nearest Iqea. You are to develop this application. You are asked to process two types of queries:

- new Iqea shop has been opened in the city with coordinates (x, y) ;
- customer wants to know the distance to the nearest already opened Iqea shop from his city located in a cell with coordinates (x, y) .

Pay attention that customer can move only by roads and can never leave Gridland on his way to the shop.

【输入格式】

The first line contains one integer n — number of cities in Gridland ($1 \leq n \leq 300000$). The following n lines contain two integers x_i and y_i — coordinates of cell that contains i -th city ($1 \leq x_i, y_i \leq 300000$). No two cells coincide. Cells form connected area, complement of this area is also connected.

The following line contains single integer q — number of queries ($0 \leq q \leq 300000$). Following q lines contain queries, i -th line consists of three integers t_i , x_i and y_i ($t_i \in 1,2$, $1 \leq x_i, y_i \leq 300000$). If $t_i = 1$, new Iqea shop has been opened in the city with coordinates (x_i, y_i) . It's guaranteed that there was no Iqea shop in this city before. If $t_i = 2$, you are to find the distance to the nearest already opened Iqea shop from the city with coordinates (x_i, y_i) . It's guaranteed that in queries of both types cell (x_i, y_i) belongs to Gridland.

【输出格式】

For every query of second type output single integer — minimum distance to the nearest Iqea shop. If no Iqea shop has been already opened, output -1 .

【测试样例】

样例输入	样例输出
7 1 2 1 3 2 3 3 1 3 2 3 3 4 2 5 2 3 2 1 4 2 2 1 2 1 3 3 2 2 3	-1 5 1

样例输入	样例输出
6 1 1 1 2 1 3 2 1 2 2 3 1 4 1 3 1 2 1 2 1 2 2 2 1 3	3 2